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Rasch First? Factor First?

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BACKGROUND

- The Rasch model and its extensions have become popular tools for assessing the psychometric properties of patient-reported outcome (PRO) instruments.
- Unidimensionality is a key assumption of the Rasch model. To gain the advantages of Rasch modeling, it is important that this assumption has not been violated.
- There has been much debate about whether to use factor analysis as a first step to assess dimensionality or whether to use the Rasch model directly to identify items that do not fit a unidimensional model.
- Existing literature provides limitations to both methods as follows:
  - Factor analysis may identify too many factors: Factor analysis usually reports items clustering at different performance levels (item difficulty) as different dimensions; thus, spurious factors (underlying concepts) may be identified.[1]
  - The Rasch model may not identify all relevant factors: The Rasch model constructs an interval variable from the dominant dimension in the data. This dominant dimension may be a hybrid of two or more factors (underlying concepts).[2] Item-fit statistics may not be sensitive in detecting off-dimension items.

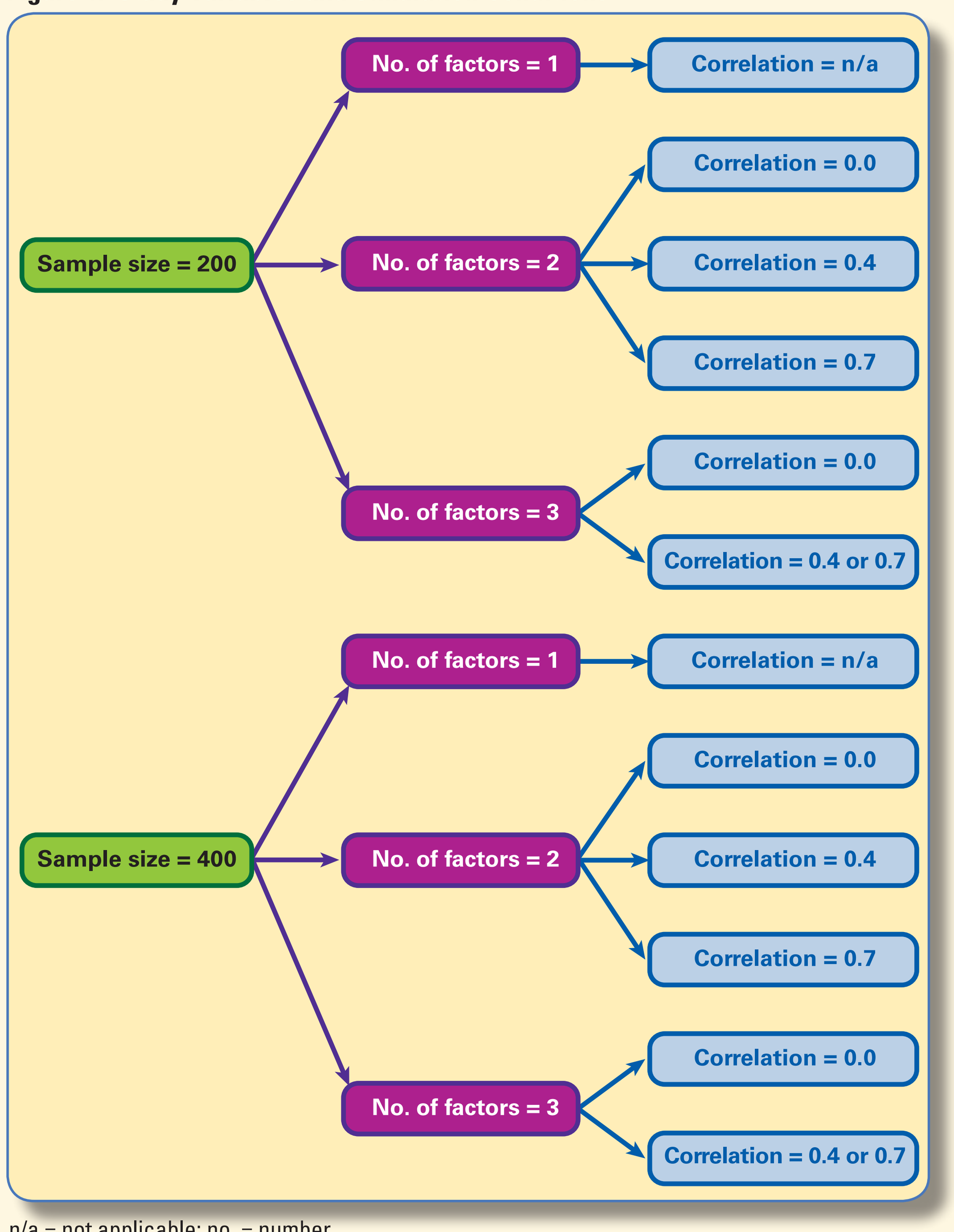
OBJECTIVE

- To compare the use of factor analysis with the use of Rasch modeling to examine the assumption of unidimensionality and provide recommendations for future application of these methods.

METHODS

- Simulated data that represent a typical PRO instrument were generated based on the following variables: sample size, number of factors, and correlation among factors.
- Figure 1 shows the design of the data simulation across the variables and the number of different conditions that it generated.

Figure 1. Study Variables and Simulation Conditions



n/a = not applicable; no. = number.

- Ten simulated data sets were generated for each of the 12 conditions, for a total of 120 data sets (Figure 1).
- Each simulated data set contained 15 items, and all items had five response categories.
- Exploratory factor analysis (EFA) was conducted using M-plus.[3]
  - The number of factors identified by the EFA was determined by examining the eigenvalues; the model goodness-of-fit statistics, including comparative fit index (> 0.95), root mean square error of approximation (< 0.05), and squared root mean residual (< 0.05); and the factor loadings.
- Rasch model analysis was conducted using RUMM2030.[4]
  - Individual item fit was examined by item-fit residual (< –2.5 or > 2.5 flagged as misfit) and item-fit chi-square ($P < 0.001$ flagged as misfit), and unidimensionality was examined by the principal component analysis of the residuals (residual variance accounted > 20%).
- Table 1 shows the detailed parameters used for each of the variables for the data simulation.

Table 1. Parameters for the Data Simulation

Model	Andrich's rating scale model	
Response category	5	
Sample sizes	200 and 400	
Number of factors	1 *(control)*, 2, and 3	
Number of items	1 factor = 15 *(control)*	
	2 factors = 9, 6	
	3 factors = 7, 4, 4	
Correlation among factors	1 factor = n/a *(control)*	
	2 factors = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	*(control)*
	2 factors = $\begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$	*(moderately correlated)*
	2 factors = $\begin{pmatrix} 1 & 0.7 \\ 0.7 & 1 \end{pmatrix}$	*(highly correlated)*
	3 factors = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	*(control)*
	3 factors = $\begin{pmatrix} 1 & 0.4 & 0.4 \\ 0.4 & 1 & 0.7 \\ 0.4 & 0.7 & 1 \end{pmatrix}$	*(moderately/highly correlated)*
Item locations	Odd-number items *normal* (–0.7, 0.7)	
	Even-number items *normal* (0.7, 0.7)	
Difference between the threshold parameters	*Uniform* (0, 1)	
Theta parameters	*Normal* (0, 1)	

Table 2. EFA: Percentages of Simulated Sets With Eigenvalues Greater Than 1

Simulated Sample Size	Number of Simulated Factors	Simulated Correlations	One Eigenvalue > 1	Two Eigenvalues > 1	Three Eigenvalues > 1
200	1	n/a	90%	10%	0%
200	2	0.0	0%	100%	0%
200	2	0.4	0%	100%	0%
200	2	0.7	0%	100%	0%
200	3	0.0	0%	0%	100%
200	3	0.4-0.7	0%	30%	70%
400	1	n/a	100%	0%	0%
400	2	0.0	0%	100%	0%
400	2	0.4	0%	100%	0%
400	2	0.7	0%	100%	0%
400	3	0.0	0%	0%	100%
400	3	0.4-0.7	0%	30%	70%

Note: Percentages are based on a total of 10 simulated data sets under each condition.

Note: Numbers in blue indicate results that align with the number of factors that were simulated; numbers in red indicate results that do not align with the number of factors that were simulated.

Table 3. EFA: Percentages of Simulated Sets With Model Goodness-of-Fit Indices Meeting the Criteria for One-, Two-, or Three-Factor Solution

Simulated Sample Size	Number of Simulated Factors	Simulated Correlations	1-Factor Solution	2-Factor Solution	3-Factor Solution
200	1	n/a	100%	--	--
200	2	0.0	0%	100%	--
200	2	0.4	0%	100%	--
200	2	0.7	0%	100%	--
200	3	0.0	0%	0%	100%
200	3	0.4-0.7	0%	40%	60%
400	1	n/a	100%	--	--
400	2	0.0	0%	100%	--
400	2	0.4	0%	100%	--
400	2	0.7	0%	100%	--
400	3	0.0	0%	0%	100%
400	3	0.4-0.7	0%	20%	80%

Note: Percentages are based on total of 10 simulated data sets under each condition

Note: Numbers in blue indicate results that align with the number of factors that were simulated; numbers in red indicate results that do not align with the number of factors that were simulated.

Note: Based on the parsimony principal, the counts did not proceed to higher factor solutions when the parsimony model met the criteria.

Table 4. Rasch Model Analysis: Item-Level Misfit and Residual Principal Component Analysis

Simulated Sample Size	Number of Simulated Factors	Simulated Correlations	Items With Misfit Residuals Residuals ≥|2.5|, Mean (Min-Max)	Items With Item Misfit Chi-square $P < 0.001$, Mean (Min-Max)	Residual Principal Component Variance Accounted > 20%, Mean (Min-Max)
200	1	n/a	0 (0-0)	0 (0-0)	0 (0-0)
200	2	0.0	0.7 (0-2)	0.9 (0-3)	1.0 (1-1)
200	2	0.4	0.2 (0-1)	0.2 (0-1)	1.0 (1-1)
200	2	0.7	0 (0-0)	0 (0-0)	0 (0-0)
200	3	0.0	0.3 (0-1)	0.3 (0-1)	0.9 (0-1)
200	3	0.4-0.7	0 (0-0)	0.1 (0-1)	0.5 (0-1)
400	1	n/a	0 (0-0)	0 (0-0)	0 (0-0)
400	2	0.0	1.2 (0-3)	1.4 (0-5)	1.0 (1-1)
400	2	0.4	0.1 (0-1)	0 (0-0)	1.0 (1-1)
400	2	0.7	0 (0-0)	0 (0-0)	0 (0-0)
400	3	0.0	2.2 (0-6)	2.1 (0-6)	1.0 (1-1)
400	3	0.4-0.7	0.5 (0-2)	(0-0)	0.8 (0-1)

min = minimum; max = maximum.

RESULTS

Exploratory Factor Analysis Key Results

- Table 2 presents the number of eigenvalues greater than 1 across the simulation conditions. Table 3 presents the number of simulated sets with goodness-of-fit indices meeting the criteria for one-, two-, or three-factor solutions.
- Results for sample sizes of 200 and 400 with corresponding factors and correlations were very similar.
- Generally, EFA correctly identified the number of factors.
  - Ten simulations were conducted for a sample size of 200 with one factor, and only one set had two eigenvalues greater than 1. This finding could be attributed to the item location factor. However, this spurious factor was not detected when the sample size was 400. For all one-factor sets, goodness-of-fit indices of one-factor solution met the criteria for adequate fit.
  - EFA correctly identified two factors for all two-factor simulations, regardless of sample size and magnitude of correlation.
  - EFA correctly identified three factors for all 20 noncorrelated three-factor simulated data sets. Of the 20 correlated three-factor solutions conducted, six sets identified two factors based on the eigenvalues-greater-than-1 rule, and six sets identified two factors based on goodness-of-fit indices (4 when sample size = 200; 2 when sample size = 400).

Rasch Model Key Results

- Table 4 presents the Rasch model results across the 12 simulation conditions. Three methods were used to evaluate the detection of the number of true factors (item-fit residuals, item-fit chi-square, principal component analysis of the residuals).
- More misfitting items were identified in the simulations with a sample size of 400 than sample size of 200.
- Generally, the Rasch model underestimated the number of items that should be associated with a separate factor.
- All 20 one-factor Rasch model simulations resulted in no misfitting items, and none of the principal components based on the residuals accounted for more than 20% of the residual variance.
- The two-factor Rasch model simulations yielded the following results:
  - The two-factor Rasch simulations with no correlation between factors resulted in zero to five items with identified misfit. Simulated correlations of 0.4 resulted in zero or one misfitting items.
  - Simulated correlations of 0.7 resulted in no misfitting items.
  - Simulated correlations of 0.0 or 0.4 resulted in one principal component accounting for more than 20% of the residual variance.
  - Correlations of 0.7 resulted in no residual principal component accounting for more than 20% of the residual variance.
- Similarly, for the three-factor simulations, the Rasch model identified fewer misfitting items for correlated factors and also detected no residual principal component accounting for more than 20% of the residual variance.

DISCUSSION

- In almost all conditions, EFA correctly identified the number of factors; the exception was simulations with three correlated factors.
  - When there were three correlated factors, EFA identified two factors an average of 30% of the time where one factor comprised the largest number of items (7 items) and the other factor comprised the remaining items.
  - The item location effect was barely detected when there was only one factor (1 of 10 data sets when sample size = 200).
  - The results showed that eigenvalue and goodness-of-fit indices performed well at identifying underlying concepts, but the best strategy was to use both criteria in combination with factor loadings. This method identified not only the number of factors, but also the items loading on each of the factors.
- The use of item-fit statistics to identify off-dimension items in the Rasch model analysis yielded inconsistent results. The number of items identified as misfitting varied from zero to six when there was more than one factor.
  - The detection of off-dimension items was better with the larger sample size (N = 400) but was worse when the factors correlated highly.
  - When correlation between two factors was as high as 0.7, no items were identified as misfitting. This was not surprising, given that the Rasch model fit the data with the hybrid of the correlated factors as the underlying dimension.
  - In most situations, however, the principal component analysis of the residuals suggested that the items were not unidimensional.

CONCLUSIONS

- The decision to conduct Rasch model analysis first or factor analysis first depends on the rationale and objective of the PRO measure under development.
- Rasch model analysis can be conducted first if the objective is to create a unidimensional score scale that summarizes the items as a whole without concern for the underlying concept or concepts.
  - The unidimensional interval scale is then a combination of the underlying concepts predominately being assessed by the final set of items.
- Factor analysis can be conducted first if the objective is to explore the underlying concepts that the items are measuring, with the potential to create meaningful subscale scores.
  - In this case, factor analysis enables researchers to identify the concepts, as well as the set of items that assesses each concept and then determine the next development steps (e.g., to focus on particular dimensions, develop additional items for dimensions with few items).

REFERENCES

1. Linacre JM. Rasch analysis first or factor analysis first? Rasch Measurement Transactions. 1998;11(4):603.
2. Wright BD. Local dependency, correlations and principal components. Rasch Measurement Transactions. 1996;10(3):509-11.
3. Muthén LK, Muthén B. Mplus user's guide, 3rd ed. Los Angeles: Muthén and Muthén; 1998-2004.
4. Andrich D, Sheridan B, Lou G. RUMM2030. Perth, Australia: RUMM Laboratory; 2009.

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